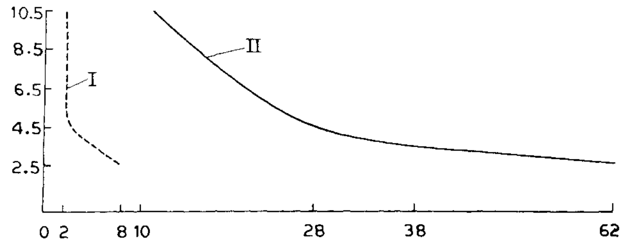

Fig. 2 The eigenvalues λ vs θ_0. The scale for the curve I is one unit = 0.1 deg and for the curve II is one unit = 1 deg.

Since $Re(\lambda) > 1$ and $\lambda \neq 2$ and the shear stress vanishes at 0, in what follows we confine ourselves to all real values of $\lambda > 2$. It can be seen easily that, for any integer value of $\lambda \geq 3$, Eq. (17) is satisfied identically by any value of θ_0. In fact, Batchelor's solution corresponds to $\lambda = 3$. Our analysis, however, reveals that it also is possible to have a dividing streamline with arbitrary θ_0 for any integer value of $\lambda > 3$. Furthermore, when $\lambda > 2$ and is not an integer, θ_0 is determined by solving Eq. (17) numerically. For a given value of λ, Eq. (17) is a transcendental equation in θ_0, which admits of infinite number of roots. The two curves in Fig. 2 correspond to values of λ for the first two positive roots. It can be seen that, as λ increases, θ_0 decreases. We thus find that an infinite number of solutions is possible for flow near a dividing streamline. The uniqueness of the solution is perhaps achieved by considering the circumstances outside the region in which each of the preceding solutions is valid.

We have seen from Eq. (17) that any integer value of $\lambda \geq 3$ is an eigenvalue regardless of the value of θ_0. It is of interest to see what happens when $\lambda = 2$. Schubert investigated this case also and arrived at the erroneous conclusion that $\lambda = 2$ is an eigenvalue when $\theta_0 = \pi/4$. We now show that $\lambda = 2$ does not correspond to a solution with a dividing streamline. In this case, the solutions of $\nabla^4 \psi = 0$ in the two regions are

$$\psi_i = \gamma^2 (A_i \cos 2\theta + B_i \sin 2\theta + C_i \theta + D_i) \quad (i = 1,2) \tag{18}$$

When Eq. (18) is used in the boundary conditions (4-8) and the constants A_i, B_i, C_i, and D_i $(i = 1,2)$ are eliminated, the following equation is obtained:

$$(1 - \cos 2\theta_0)(2 \log\gamma \cdot \cos 2\theta_0 - 1 + \cos 2\theta_0) = 0 \tag{19}$$

which can hold only when $\cos 2\theta_0 = 1$, leading to $\theta_0 = n\pi$, $n = 0, 1, 2, \ldots$. Thus when $\lambda = 2$ no solution with a dividing streamline is possible, contrary to the conclusion of Schubert. This result also could have been expected from physical considerations, since, when $\psi \sim \gamma^2 f_2(\theta)$, the expression for the shear stress becomes independent of γ and hence cannot vanish at the point 0 (Fig. 1), which contradicts the fact that 0 is a point of zero skin friction.

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References

[1] Moffatt, H. K., "Viscous and Resistive Eddies near a Sharp Corner," *Journal of Fluid Mechanics*, Vol. 18, 1964, pp. 1-18.

[2] Lugt, H. J. and Schwiderski, E. W., "Flows around Dihedral Angles," *Proceedings of the Royal Society, (London)*, Vol. A285, 1965, pp. 382-399.

[3] Batchelor, G. K., *An Introduction to Fluid Dynamics*, Cambridge University Press, London, 1967, pp. 226-227.

[4] Schubert, G., "Flow near the Intersection of a Wall and a Dividing Streamline," *AIAA Journal*, Vol. 6, March 1968, pp. 549-550.

Simplified Formulas for Lift and Moment in Unsteady Thin Airfoil Theory

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RECENTLY, Williams[1] presented results for the pressure distribution, lift, and moment of an arbitrary oscillatory motion in subsonic thin airfoil theory. He showed that they could all be obtained from a knowledge of the loading distributions for plunging (heaving) motion and pitching about the leading edge. He expressed his results in a coordinate system originating at the leading edge, and found the lift to depend only on the plunging loading, while the moment depends on both the plunging and pitching loading.

The purpose of this Note is to point out that the formula for the moment takes a remarkably simple form when reformulated in a coordinate system originating at the airfoil center, with pitching taken about that center. The moment then depends only on the loading distribution for pitching, just as the lift depends only on the loading distribution for plunging.

Williams uses a set of dimensionless variables in which length is measured in chord lengths c, velocities in stream speed U, time in units of c/U, loading P in units of ρU^2, lift in units of $\rho U^2 c$, and moment in units of $\rho U^2 c^2$. The dimensionless (reduced) frequency k is related to the frequency ν by $k = \nu c/U$. He denotes by P_h the loading induced by the plunging downwash ik, and by P_α the loading induced by the pitching downwash (about the leading edge) $1 + ikx$. In those terms, he finds the lift for an arbitrary downwash $w(x)$ to be his Eq. (18):

$$L = \int_0^1 w(1-x) P_h(x) \frac{dx}{ik} \tag{1}$$

The moment about the leading edge (nose down) is given by his Eq. (19):

$$M = -\int_0^1 w(1-x) \left[(1+ik) P_h(x) - ikP_\alpha(x) \right] \frac{dx}{k^2} \tag{2}$$

In computing the aerodynamic loads on oscillating airfoils, it is often convenient to use a coordinate system originating at the airfoil center, normalized by the half-chord. If this variable is called $\bar{x}$, it is related to x by

$$\bar{x} = 2x - 1 \qquad -1 < \bar{x} < 1 \tag{3}$$

Any function of x can be expressed as a function of $\bar{x}$ by

$$f(x) = f[(\bar{x}+1)/2] = \bar{f}(\bar{x}) = \bar{f}(2x-1) \tag{4a}$$

$$f(1-x) = \bar{f}(2-2x-1) = \bar{f}(1-2x) = \bar{f}(-\bar{x}) \tag{4b}$$

We now define $P^{(0)}$ as the loading induced by unit plunging downwash, so that

$$P^{(0)}(x) = \bar{P}^{(0)}(\bar{x}) = P_h(x)/ik \tag{5}$$

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Similarly, we define $P^{(1)}$ to be the loading induced by a plunging downwash $\bar{x}$. Since P_α corresponds to $1+ikx = 1+ik(\bar{x}+1)/2$, we have

$$P_\alpha(x) = (1+ik/2)\bar{P}^{(0)}(\bar{x}) + ik\bar{P}^{(1)}(\bar{x})/2 \tag{6}$$

Therefore,

$$k^{-2}[(1+ik)P_h - ikP_\alpha] = k^{-2}[(1+ik)ik\bar{P}^{(0)} - ik(1+ik/2)\bar{P}^{(0)} - (ik)^2\bar{P}^{(1)}/2] = [\bar{P}^{(1)} - \bar{P}^{(0)}]/2 \tag{7}$$

Use of Eqs. (3, 4b, and 5) enable us to write Eq. (1) as

$$L = \frac{1}{2}\int_{-1}^{1} \bar{w}(-\bar{x})\bar{P}^{(0)}(\bar{x})\,\mathrm{d}\bar{x} \tag{8}$$

Similarly, Eqs. (3, 4b, and 7) transform Eq. (2) to

$$M = -\frac{1}{4}\int_{-1}^{1} \bar{w}(-\bar{x})[\bar{P}^{(1)}(\bar{x}) - \bar{P}^{(0)}(\bar{x})]\mathrm{d}\bar{x} = -\frac{1}{4}\int_{-1}^{1} \bar{w}(-\bar{x})\bar{P}^{(1)}(\bar{x})\,\mathrm{d}\bar{x} + \frac{1}{2}L \tag{9}$$

where Eq. (8) has been used in the last step. Finally, the nose-down moment about the center is

$$M_0 = M - \frac{1}{2}L = -\frac{1}{4}\int_{-1}^{1} \bar{w}(-\bar{x})\bar{P}^{(1)}(\bar{x})\,\mathrm{d}\bar{x} \tag{10}$$

Equations (8) and (10) are the present versions of Williams' results. The first gives the lift as an integral involving the loading induced by unit plunging downwash $\bar{P}^{(0)}$, as Williams found. The second gives the nose-down moment about the airfoil center as an integral involving only $\bar{P}^{(1)}$; the loading induced by a pitching downwash whose chordwise dependence is $\bar{x}$. In both cases, the arbitrary motion of the airfoil is represented by the downwash $\bar{w}(\bar{x})$, which appears in the integrals as $\bar{w}(-\bar{x})$. In the present centered coordinate system, then, not only are lift and moment for arbitrary motion determined only by the loadings for plunging and pitching, but the lift depends only on the plunging loading and the moment only on the pitching loading. This seems to be a useful reformulation of Williams' results.

We also note that for unit downwash, Eq. (10) gives

$$M_0^{(0)} = -\frac{1}{4}\int_{-1}^{1} \bar{P}^{(1)}(\bar{x})\,\mathrm{d}\bar{x} = -\frac{1}{2}L^{(1)} \tag{11}$$

by the definition of $\bar{P}^{(1)}$. This relation between the moment due to plunging and the lift due to pitching is well-known, and corresponds to Williams' more complicated Eq. (20).

It is easy to verify that Eqs. (8) and (10) give correct results when applied to the classical cases of single airfoils oscillating in incompressible flow for the various types of downwash. The same is true of in-phase oscillations of unstaggered airfoil cascades.[2]

References

[1] Williams, M. H., "Exact Solutions in Oscillating Airfoil Theory," *AIAA Journal*, Vol. 15, June 1977, pp. 875-877.

[2] Kemp, N. H. and Ohashi, H., "Forces on Unstaggered Airfoil Cascades in Unsteady In-Phase Motion," *Journal of Fluids Engineering*, Vol. 98, Sept. 1976, pp. 521-530.

Nonuniform Propagation of Sonic Discontinuities Through Thermally Conducting Gases

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Introduction

SHOCK formation is a feature of ideal gasdynamics. The basic equations are hyperbolic and nonlinear. The motion of a medium is governed by a system of equations such that an initially smooth wave front may steepen to form a compressive shock. Using the theory of singular surfaces, which was proposed and developed by Thomas[1] to study the growth of discontinuities in the continuum mechanics, Thomas,[2] Kaul,[3] Nariboli,[4,5] and Shankar[6,7] have studied the propagation of weak discontinuities through different media, under the assumption that the medium in front of the propagating surface is uniform and at rest. If the medium ahead is moving, then it can be shown[8] that the wave propagation is anisotropic. In order to study anisotropic wave propagation, Lighthill[9] has developed an elegant method which essentially involves the evaluation of the Fourier integrals by the stationary phase method and gives the asymptotic features of the solution. Numerous applications of this method followed. Ludwig[10] and Duff[11] further generalized and developed this technique.

The mathematical theory of geometric optics (ray theory) of Luneberg[12] was found useful by Bazer[13] in the investigation of the propgation of weak discontinuities. Nariboli[14] combined the theory of singular surfaces and Luneberg's method of geometric optics to study the propagation of weak discontinuities in nonlinear anisotropic media. Using this combination, he integrated the equation of growth of discontinuities in a simple and straightforward manner. Following Nariboli,[14] Upadhyay[15] obtained the growth equation for sonic discontinuities propagating through thermally conducting gases, but he did not discuss the growing or decaying tendency.

Recently, Elcrat[16] studied the nonuniform propagation of sonic discontinuities in an unsteady flow of a perfect gas. In order to integrate the growth equations, he transformed them to an equation along the bicharacteristic curve in the characteristic manifold. While doing so, he arrived at an ordinary differential equation which was solved completely, and the criterion for decay or "blow up" was obtained. In the present Note, following Elcrat,[16] we shall derive and discuss the solutions of fundamental differential equations for nonuniform propagation of sonic discontinuities through the thermally conducting gases. We shall also find the criterion for decay or "blow up" for sonic discontinuities.

II. Inviscid Nonconducting Gas with Finite Thermal Conductivity

The set of nonlinear differential equations governing the inviscid nonconducting gas with finite thermal conductivity is given by[15],

$$\frac{\partial \rho}{\partial t} + v_i,\rho_i + \rho v_{i,i} = 0 \tag{1}$$

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